NASA TECHNICAL MEMORANDUM

NASA TM X-1303

NASA TM X-1303

GPO PRICE $ ______

CFSTI PRICE(S) $ ______

Hard copy (HC) ______

Microfiche (MF) ______

ff 653 July 65

N66 38863

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by Fred S. Sidransky

Lewis Research Center

Cleveland, Ohio

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • OCTOBER 1966

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SUMMARY

The method of characteristics is used to present general compatibility relations for nonsteady liquid flow or water-hammer theory which permit the investigation of the dynamics of the flow under diverse conditions; that is, for a liquid flowing in a variable area conduit affected by heat addition, by gravity, by friction forces, and by centrifugal forces such as in a mixed-flow radial vane pump. The compatibility equations are presented in finite difference form, and numerical-graphical methods for their solution are indicated.

INTRODUCTION

In rocket engine and Rankine cycle systems, the analysis of nonsteady liquid flow is often complicated by the transmission of the fluid in a conduit with a gradually varying cross-sectional area. In addition, the fluid may be affected by heat addition and a number of forces such as gravity, dissipative, and centrifugal (e. g., as in a pump). Hence, a general method is required which will allow the investigation of fluid transients under such diverse conditions.

Analyses for nonsteady liquid flow have been presented in a number of papers for particular conditions; that is, for a tapered line (ref. 1), for friction (ref. 2), and for a pump as a lumped component in a conduit (ref. 3).

In this report the theory of nonsteady liquid flow as developed in reference 4 will be extended so that the dynamics of a liquid in a conduit with a gradually varying cross-sectional area, undergoing heat addition, and affected by the previously mentioned forces may be analyzed. Applications will be outlined for nonsteady liquid flow with heat addition, for tapered conduits with gravity and dissipative forces, and for the internal dynamics of a mixed flow pump with radial vanes.

ANALYSIS

Derivation of Fundamental Compatibility Relations

Continuity. - The continuity equation (ref. 5) is given by

$$\frac{\partial}{\partial t}(\rho A) + \frac{\partial}{\partial \xi}(\rho u A) = 0 \tag{1}$$

and if $\frac{\partial A}{\partial t} = 0$, equation (1) may be expanded to

$$A \frac{\partial \rho}{\partial t} + \left(\frac{\partial \rho}{\partial \xi} u + \frac{\partial u}{\partial \xi} \rho\right) A + \frac{\partial A}{\partial \xi} \rho u = 0 \tag{2}$$

The term $\frac{\partial A}{\partial t}$, a function of the elasticity of the conduit, is omitted since it is usually negligibly small compared to the variation of area with distance along the conduit $\frac{\partial A}{\partial \xi}$. If equation (2) is divided by ρA, then

$$\frac{1}{\rho}\frac{\partial \rho}{\partial t} + \frac{1}{\rho} u \frac{\partial \rho}{\partial \xi} + \frac{\partial u}{\partial \xi} + u \frac{\partial (\ln A)}{\partial \xi} = 0 \tag{3a}$$

and since

$$P = f(\rho, s) \tag{3b}$$

equation (3) may be expressed by

$$\frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\frac{\partial P}{\partial t} + \frac{\partial \rho}{\partial s}\frac{\partial s}{\partial t}\right) + \frac{1}{\rho}\left(\frac{\partial \rho}{\partial P}\frac{\partial P}{\partial \xi} + \frac{\partial \rho}{\partial s}\frac{\partial s}{\partial \xi}\right)u + \frac{\partial u}{\partial \xi} + u \frac{\partial (\ln A)}{\partial \xi} = 0 \tag{4}$$

Assuming that $\frac{\partial \rho}{\partial s}$ is negligibly small for a liquid and using the definition for the acoustic velocity, namely,

$$\left(\frac{\partial \rho}{\partial P}\right)_s = \frac{1}{a^2} \tag{5}$$

result in

$$\frac{1}{\rho a^2}\frac{\partial P}{\partial t} + \frac{1}{\rho a^2}\frac{\partial P}{\partial \xi} u + \frac{\partial u}{\partial \xi} + u\frac{\partial(\ln A)}{\partial \xi} = 0 \tag{6}$$

Inasmuch as the pressure P is expressed in force per unit area and the density ρ in mass per unit volume and the head in the units of pressure divided by the specific weight of the fluid in the Earth's gravitational field, the following relation may be deduced

$$\frac{P - P_o}{\rho} = g_c H \tag{7}$$

where P_o is the datum pressure, a constant. In view of equation (7) and the assumption that the variation of the density has a negligibly small effect on the head (i.e., ρ = const.), equation (6) may be presented by

$$\frac{g_c}{a^2}\frac{\partial H}{\partial t} + \frac{g_c}{a^2} u \frac{\partial H}{\partial \xi} + \frac{\partial u}{\partial \xi} + u\frac{\partial(\ln A)}{\partial \xi} = 0 \tag{8}$$

Multiplying by a^2/g_c and transposing some terms result in

$$\frac{\partial H}{\partial t} + u\frac{\partial H}{\partial \xi} + \frac{a^2}{g_c}\frac{\partial u}{\partial \xi} = -\frac{ua^2}{g_c}\frac{\partial(\ln A)}{\partial \xi} \tag{9}$$

<u>Momentum</u>. - Consider the momentum equation given by (ref. 5)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial \xi} = -\frac{1}{\rho}\frac{\partial P}{\partial \xi} + f \tag{10}$$

Multiplying equation (10) by a/g_c and considering equation (7) give

$$\frac{a}{g_c}\frac{\partial u}{\partial t} + \frac{a}{g_c} u \frac{\partial u}{\partial \xi} + a\frac{\partial H}{\partial \xi} = \frac{a}{g_c} f \tag{11}$$

Subtracting equation (11) from equation (9) yields

$$\frac{\partial}{\partial t}\left(H - \frac{a}{g_c} u\right) + (u - a)\frac{\partial H}{\partial \xi} - \frac{a}{g_c}\left(u\frac{\partial u}{\partial \xi} - a\frac{\partial u}{\partial \xi}\right) = -\frac{ua^2}{g_c}\frac{\partial(\ln A)}{\partial \xi} - \frac{a}{g_c} f \tag{12}$$

if a/g_c is a constant. By further algebraic manipulation, equation (12) can be presented in the form

$$\frac{\partial}{\partial t}\left(H - \frac{a}{g_c} u\right) + (u - a)\frac{\partial}{\partial \xi}\left(H - \frac{a}{g_c} u\right) = -\frac{ua^2}{g_c}\frac{\partial(\ln A)}{\partial \xi} - \frac{a}{g_c} f \qquad (13)$$

Adding equations (9) and (11) yields another expression, namely,

$$\frac{\partial}{\partial t}\left(H + \frac{a}{g_c} u\right) + (u + a)\frac{\partial}{\partial \xi}\left(H + \frac{a}{g_c} u\right) = -\frac{ua^2}{g_c}\frac{\partial(\ln A)}{\partial \xi} + \frac{a}{g_c} f \qquad (14)$$

Now let

$$H + \frac{a}{g_c} u = C_x \qquad (15)$$

and

$$H - \frac{a}{g_c} u = C_y \qquad (16)$$

and define two directional derivatives as follows:

$$\frac{\delta^+}{\delta t} = \frac{\partial}{\partial t} + (u + a)\frac{\partial}{\partial \xi} \qquad (17)$$

and

$$\frac{\delta^-}{\delta t} = \frac{\partial}{\partial t} + (u - a)\frac{\partial}{\partial \xi} \qquad (18)$$

With the help of equations (15) to (18), equations (13) and (14) may be expressed by

$$\frac{\delta^- C_y}{\delta t} = -\frac{ua^2}{g_c}\frac{\partial(\ln A)}{\partial \xi} - \frac{a}{g_c} f \qquad (19)$$

$$\frac{\delta^+ C_x}{\delta t} = -\frac{ua^2}{g_c}\frac{\partial(\ln A)}{\partial \xi} + \frac{a}{g_c} f \qquad (20)$$

In the terminology of the method of characteristics (ref. 5, vol. 1, appendix A), equations (19) and (20) can be considered as right running and left running compatibility equations respectively, and their characteristic directions in the time-distance plane are

$$\frac{d\xi}{dt} = u - a \tag{21}$$

along the right running characteristic and

$$\frac{d\xi}{dt} = u + a \tag{22}$$

along the left running characteristic. The similarity of these results and those of reference 4 is evident.

If the flow velocity is very small compared to the acoustic velocity, equations (21) and (22) become simply

$$\frac{d\xi}{dt} = -a \tag{23}$$

$$\frac{d\xi}{dt} = a \tag{24}$$

If the two terms appearing to the right of equations (19) and (20) are omitted or set equal to zero, then

$$\frac{\delta^+ C_y}{\delta t} = 0 \tag{25a}$$

and

$$C_y = H - \frac{a}{g_c} u \tag{25b}$$

is the right running compatibility equation and

$$C_x = H + \frac{a}{g_c} u \tag{26}$$

is the left running compatibility relation. These equations are the more familiar water-hammer relations.

Energy. - The energy equation gives rise to a third compatibility relation and characteristic direction. This may be seen by considering the energy equation in a conduit as expressed by (ref. 5)

$$q\rho A = \frac{\partial}{\partial t}\left[\rho A\left(i + \frac{u^2}{2}\right)\right] + \frac{\partial}{\partial \xi}\left[\rho u A\left(i + Pv + \frac{u^2}{2}\right)\right] \tag{27}$$

which after some manipulation becomes

$$q\rho A = \left(i + \frac{u^2}{2}\right)\left[\frac{\partial(\rho A)}{\partial t} + \frac{\partial(\rho u A)}{\partial \xi}\right] + \frac{\partial(uAP)}{\partial \xi} + \rho A\left(\frac{\partial i}{\partial t} + u\frac{\partial i}{\partial \xi}\right) + \rho A\left[\frac{\partial}{\partial t}\left(\frac{u^2}{2}\right) + u\frac{\partial}{\partial \xi}\left(\frac{u^2}{2}\right)\right] \tag{28}$$

By the definition of the co-moving or substantial derivative, namely,

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{d\xi}{dt}\frac{\partial}{\partial \xi} \tag{29a}$$

or

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + u\frac{\partial}{\partial \xi} \tag{29b}$$

and, in view of equation (1), equation (28) may be presented by

$$P\frac{\partial(uA)}{\partial \xi} + uA\frac{\partial P}{\partial \xi} + \rho A\frac{Di}{Dt} + \rho A u\frac{Du}{Dt} = q\rho A \tag{30}$$

From continuity, the following relation is obtained:

$$\frac{\partial(uA)}{\partial \xi} = A\rho\frac{D\left(\frac{1}{\rho}\right)}{Dt} \tag{31}$$

where $\frac{\partial A}{\partial t}$ is omitted. Hence, instead of equation (30),

$$\rho AP \frac{D\left(\frac{1}{\rho}\right)}{Dt} + \rho Au\left(\frac{1}{\rho}\frac{\partial P}{\partial \xi} + \frac{Du}{Dt}\right) + \rho A \frac{Di}{Dt} = \rho Aq \tag{32}$$

is obtained. In view of equation (10), equation (32) may be transformed to

$$P \frac{D\left(\frac{1}{\rho}\right)}{Dt} + \frac{Di}{Dt} + uf = q \tag{33}$$

Since, from thermodynamics, it is known that

$$P \frac{D\left(\frac{1}{\rho}\right)}{Dt} + \frac{Di}{Dt} = T \frac{Ds}{Dt} \tag{34}$$

it follows that

$$T \frac{Ds}{Dt} = q \tag{35}$$

if the magnitude of the term uf, which represents in the negative sense the specific work done by dissipative forces per unit time, is much smaller than the value of q (i.e., the heat addition is reversible). Equation (35) is the third compatibility equation; and in view of equations (29a) and (29b), the direction of the characteristic in the time-distance plane will be given by

$$\frac{d\xi}{dt} = u \tag{36}$$

Hence, in general, a net point in nonsteady liquid flow is defined by three characteristics, the projection of which on the time-distance plane is seen in figure 1.

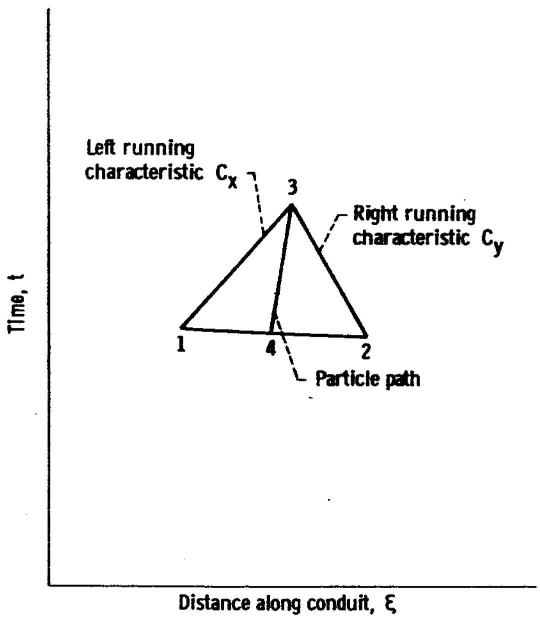

Figure 1. - General characteristic net point for nonsteady liquid flow or water hammer.

Heat addition. - Equations (15) and (16) are two equations with two unknowns, H and u, since C_x is a constant along the left running characteristic and C_y is a constant along the right running characteristic, and the acoustic velocity a is also constant. If none of the parameters of equations (15) and (16) are affected by entropy change, which can be reasonably assumed in liquids, the contribution of heat addition to the dynamic analysis of liquids may be omitted. Yet it is desirable to estimate the temperature variation of a liquid as it proceeds, for example, through a boiler or heated pump inlet so as to predict the incipience of boiling. This may be accomplished by following a particle of fixed identity in the fluid as implied in equation (35). From thermodynamics, it is given that for constant volume reversible heating

$$T\,Ds = C_v DT \tag{37}$$

Integrating equation (37) along the particle path (cf. fig. 1) results in

$$T_3 = T_4 e^{(s_3 - s_4)/C_v} \tag{38}$$

and, consequently, by equation (35)

$$T_3 = T_4 e^{\psi(t_3 - t_4)/C_v} \tag{39}$$

where

$$\psi = \frac{q}{T}$$

Evaluation of the Specific Force

The specific force f (or force per unit mass) appearing in the fundamental momentum equation (eq. (10)) is undefined. It may be equated to a specific gravity force, or a dissipative force, or a specific centrifugal force, or the sum of all these depending on the particular study to be made. In the following, the specific force f will be equated, first, to the sum of the specific gravity and specific dissipative forces and, then, in order to analyze the internal dynamics of a mixed-flow radial pump, to the sum of specific centrifugal and specific dissipative forces.

Gravity and dissipative forces. - If the gravity force per unit mass, or in other words the specific gravity force g, is opposite in direction to the assumed positive direction of the specific force f, the sum of the specific gravity and dissipative forces may be expressed by

$$f = -\frac{\lambda u^2}{2D}\frac{u}{|u|} - g \tag{40}$$

where λ is a friction coefficient and D is the hydraulic diameter of the duct. The factor $u/|u|$ ensures that the dissipative force is opposite in direction to the flow velocity. By equation (40) the compatibility equations (eqs. (19) and (20)) become

$$\frac{\delta^- C_y}{\delta t} = -\frac{ua^2}{g_c}\frac{\partial(\ln A)}{\partial \xi} + \frac{a}{g_c}\left(\frac{\lambda u^2}{2D}\frac{u}{|u|} + g\right) \tag{41}$$

$$\frac{\delta^+ C_x}{\delta t} = -\frac{ua^2}{g_c}\frac{\partial(\ln A)}{\partial \xi} - \frac{a}{g_c}\left(\frac{\lambda u^2}{2D}\frac{u}{|u|} + g\right) \tag{42}$$

or, in finite difference form,

$$C_{y_3} = C_{y_2} - \left[\frac{ua^2}{g_c}\frac{\partial(\ln A)}{\partial \xi}\right]_{23}(t_3 - t_2) + \left(\frac{a}{g_c}\frac{\lambda u^2}{2D}\frac{u}{|u|} + g\right)_{23}(t_3 - t_2) \tag{43}$$

$$C_{x_3} = C_{x_1} - \left[\frac{ua^2}{g_c}\frac{\partial(\ln A)}{\partial \xi}\right]_{13}(t_3 - t_1) - \left(\frac{a}{g_c}\frac{\lambda u^2}{2D}\frac{u}{|u|} + g\right)_{13}(t_3 - t_1) \tag{44}$$

and, for the corresponding characteristic directions,

$$\frac{\xi_3 - \xi_2}{t_3 - t_2} = -a_{23} \tag{45}$$

$$\frac{\xi_3 - \xi_1}{t_3 - t_1} = a_{13} \tag{46}$$

where the subscripts refer to net points in the characteristic network (cf. fig. 1). The subscripts 13 and 23 represent average values between net point 1 and 3 and net point 2 and 3, respectively. Clearly, equations (43) and (44) may be more easily evaluated if the variation of area with conduit length is expressed in the form

$$A = e^{h(\xi)} \tag{47a}$$

or

$$\frac{\partial(\ln A)}{\partial \xi} = \frac{\partial h(\xi)}{\partial \xi} \tag{47b}$$

where $h(\xi)$ is some easily differentiable function of conduit length.

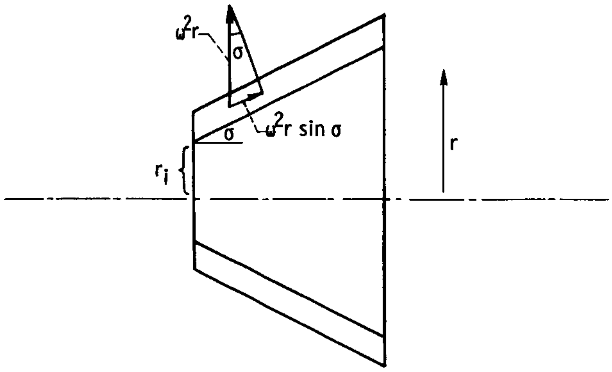

Figure 2. - Mixed-flow pump with radial vanes. Seeing that $r - r_i = \xi \sin\sigma$, it follows that $f = \omega^2\xi \sin^2\sigma + \omega^2 r_i \sin\sigma$.

Pump dynamics. - The equation of motion for a mixed-flow pump with radial vanes may be expressed by (cf. fig. 2)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial \xi} - \left(\omega^2\xi \sin^2\sigma + \omega^2 r_i \sin\sigma\right) = -\frac{1}{\rho}\frac{\partial P}{\partial \xi} \tag{48}$$

where u is the through-flow component of the flow velocity relative to the blade and σ is the angle the through-flow component makes with the axis of the pump. The terms $\omega^2\xi \sin^2\sigma + \omega^2 r_i \sin\sigma$ represent the centrifugal force per unit mass. If, in addition, friction forces are significant, then

$$f = \omega^2\xi \sin^2\sigma + \omega^2 r_i \sin\sigma - \frac{\lambda u^2}{2D}\frac{u}{|u|}$$

and equations (41) and (42) may be modified to

$$\frac{\delta^- C_y}{\delta t} = -\frac{ua^2}{g_c}\frac{\partial(\ln A)}{\partial \xi} - \frac{a}{g_c}\left(\omega^2\xi \sin^2\sigma + \omega^2 r_i \sin\sigma - \frac{\lambda u^2}{2D}\frac{u}{|u|}\right) \tag{49}$$

$$\frac{\delta^+ C_x}{\delta t} = -\frac{ua^2}{g_c}\frac{\partial(\ln A)}{\partial \xi} + \frac{a}{g_c}\left(\omega^2\xi \sin^2\sigma + \omega^2 r_i \sin\sigma - \frac{\lambda u^2}{2D}\frac{u}{|u|}\right) \tag{50}$$

or in finite difference form to

$$C_{y_3} = C_{y_2} - \left[\frac{ua^2}{g_c}\frac{\partial(\ln A)}{\partial \xi}\right]_{23}(t_3 - t_2) - \left[\frac{a}{g_c}\left(\omega^2\xi \sin^2\sigma + \omega^2 r_i \sin\sigma - \frac{\lambda u^2}{2D}\frac{u}{|u|}\right)\right]_{23}(t_3 - t_2) \tag{51}$$

$$C_{x_3} = C_{x_1} - \left[\frac{ua^2}{g_c} \frac{\partial(\ln A)}{\partial \xi}\right]_{13} (t_3 - t_1) + \left[\frac{a}{g_c}\left(\omega^2 \xi \sin^2\sigma + \omega^2 r_i \sin\sigma - \frac{\lambda u^2}{2D} \frac{u}{|u|}\right)\right]_{13} (t_3 - t_1) \tag{52}$$

If cavitation occurs in the pump (the head is less than the vapor head), a vapor region will appear in the characteristic network. The vapor head and the C_x characteristic will define the flow into the cavitation zone, and the C_y characteristic will define the flow out. The time increment and past conditions will help to estimate the augmentation in the size of the cavitation zone. Further details on the analysis of the dynamics of cavitation regions by essentially the method of characteristics may be found in reference 6.

CONCLUDING REMARKS

General compatibility relations have been presented which will permit the analysis of the dynamics of a liquid under diverse conditions. In view of their complexity, a high-speed computer should be employed in their practical application to any particular problem.

Although an electromagnetic pump has not been considered in this memorandum, it is clear that in place of the centrifugal force per unit mass term in the compatibility equations appertaining to pump dynamics, an electromagnetic force per unit mass term may be employed.

The characteristic methods of this report may be applied, for example, to an approximate estimate of the effect of the through-flow channel of a pump on flow dynamics. Further, the techniques of this report may be used to predict the dynamic formation and collapse of cavitation regions in a heated pump inlet line as well as the incipience of two-phase flow in a boiler under dynamic conditions.

Lewis Research Center,
National Aeronautics and Space Administration,
Cleveland, Ohio, June 8, 1966,
120-27-04-27-22.

APPENDIX A

SYMBOLS

A area, sq ft

a acoustic velocity, ft/sec

C_v specific heat at constant volume, ft-lb/(slug)($^{\circ}$R)

C_x parameter (see eq. (15)), ft

C_y parameter (see eq. (16)), ft

D hydraulic diameter, ft

f specific force or force per unit mass, ft/sec^2

g specific gravity force, ft/sec^2

g_c acceleration due to Earth's gravitational field, ft/sec^2

H head, ft

i internal energy, ft-lb/slug

P pressure, lb/sq ft

q heat added per unit mass per unit time, ft-lb/(slug)(sec)

r radial height (see fig. 2)

r_i inlet radial height (see fig. 2)

s specific entropy, ft-lb/(slug)($^{\circ}$R)

T temperature, $^{\circ}$R

t time, sec

u flow velocity, ft/sec

v specific volume, cu ft/slug

λ friction coefficient

ξ distance along conduit, ft

ρ density, slug/cu ft

σ angle that the relative through-flow component of velocity makes with pump axis

ψ parameter, q/T

ω angular speed, sec^{-1}

Subscripts:

o datum

1,2,3,4 refers to net point locations (see fig. 1)

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